AIEEE 2003
PHYSICS & CHEMISTRY SOLUTIONS

1. Force is $\perp^r$ to displacement $\Rightarrow$ the work done is zero

2. Since there is no deviation in the path of the charged particle, so net force due to presence of electric and magnetic field must be zero $\Rightarrow vB = qE \Rightarrow B = \frac{E}{V} = \frac{10^4}{10} = 10^3\ Wb/m^2$

3. $T \propto \sqrt{l}$

$\frac{T_1}{T_2} = \sqrt{\frac{l_1}{l_2}}$; $I \propto L^2$ $\left[\Rightarrow \frac{I_1}{I_2} = \frac{L_1^2}{L_2^2} = \frac{(2L_2)^2}{L_2^2} = 4\right] \Rightarrow \frac{T_1}{T_2} = \sqrt{4} = 2$

$\Rightarrow T_2 = \frac{T_1}{2} \Rightarrow \frac{T_2}{T_1} = \frac{1}{2}$

4. $\tau = (H)\ \tan 60^0 \quad = W.\sqrt{3}$

7. Mass $= \frac{49}{9.8} = 5kg$. When lift is moving downward, apparent weight = 5(9.8 - 5) = 5$\times$4.8 = 24 N

8. Potential $\propto$ R

R $\propto$ length $\Rightarrow$ Potential difference $\propto$ l

11. $\Delta T = \frac{40}{25\times10^{-6}\times10^{-5}} \Rightarrow \Delta T = 16^0 C$

13. $\frac{1}{\sqrt{\mu_0\varepsilon_0}} = C \Rightarrow \frac{1}{\mu_0\varepsilon_0} = C^2 \Rightarrow \left[\frac{1}{\mu_0\varepsilon_0}\right] = [C]^2$

$[C] = LT^{-1}$ or $[C]^2 = L^2T^{-2}$

14. $I = \frac{1}{2} m\ R^2$ or $M \propto t \propto R^2$

For disc X, $I_x = \frac{1}{2}(m)(R)^2 = \frac{1}{2}(\pi r^2 t).(R)^2$

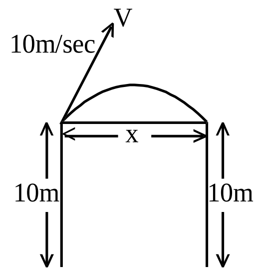
For disc Y, $I_y = \frac{1}{2}[\pi(4R)^2.t/4][4R]^2$

$\Rightarrow \frac{I_x}{I_y} = \frac{1}{(4)^3} \Rightarrow I_y = 64\ I_x$

15. $T^2 \propto R^3 \Rightarrow \left(\frac{T_1}{T_2}\right)^2 = \left(\frac{R_1}{R_2}\right)^3$

$\Rightarrow \left(\frac{T_1}{T_2}\right) = \left(\frac{R_1}{R_2}\right)^{3/2} = \left(\frac{1}{4}\right)^{3/2}$ $\Rightarrow \frac{T_2}{T_1} = (4)^{3/2} = 8$

$\Rightarrow T_2 = 8\times T_1 = 8\times5 = 40 = 40$ hours



16. Angular momentum $\propto \dfrac{1}{\text{Angular frequency}} \propto$ Kinetic energy $\Rightarrow \vec{L} = \dfrac{K.E.}{w}$

$$\frac{L_1}{L_2} = \left(\frac{K.E_1}{w_1}\right) \times \frac{w_2}{KE_2} = 4 \quad \Rightarrow L_2 = \frac{L}{4}$$

17. $\xrightarrow[\text{RMIVUXGE}]{\lambda \text{ Decreasing}}$

R $\rightarrow$ Radio waves ; M $\rightarrow$ Micro waves; I $\rightarrow$ Infra red rays; V $\rightarrow$ Visible rays; U $\rightarrow$ Ultraviolet rays; X $\rightarrow$ X rays; G $\rightarrow \gamma$ rays; C $\rightarrow$ Cosmic rays

$\Rightarrow$ γ rays has least wavelength

18. Applying the principle of conservation of linear momentum

$$(4)(u) = (v)(238) \Rightarrow v = \frac{4u}{238}$$

19. Distance between the surface of the spherical bodies = 12R - R - 2R = 9R

Force $\propto$ Mass, Acceleration $\propto$ Mass, Distance $\propto$ Acceleration

$$\Rightarrow \frac{a_1}{a_2} = \frac{M}{SM} = \frac{1}{5} \Rightarrow \frac{S_1}{S_2} = \frac{1}{5} \Rightarrow S_2 = 5S_1$$

$S_1 + S_2 = 9 \Rightarrow 6S_1 = 9 \Rightarrow S_1 = \dfrac{9}{6} = 1.5,$ $\qquad S_2 = 1.5 \times 5 = 7.5$

Note: Maximum distance will be travelled by smaller bodies due to the greater acceleration caused by the same gravitational force

21. Energy = Work done by force (F)

$$\Rightarrow \frac{1}{2} m.(50)^2 = (F)(6) \qquad \Rightarrow F = \frac{2500m}{2 \times 6}$$

For $v = 100 km/hr$ $\dfrac{1}{2}.m(100)^2 = (F)(S)$

$$\Rightarrow \frac{1}{2} m(100)^2 = \left(\frac{2500m}{2 \times 6}\right) S$$

$$\Rightarrow S = \frac{100 \times 100 \times 6 \times 2}{2500 \times 2} = 24 m$$

22. From, the question if the horizontal distance is none other than the horizontal range on the level of the roof of building

$$\text{Range} = \frac{u^2 \sin 2\theta}{g} = \frac{(10)^2 \sin(2 \times 30)}{g} = \frac{10 \times 10 \times \sqrt{3}}{2 \times 10} = 8.66$$

24. $[\text{momentum}] = [M][L][T^{-1}] = [MLT^{-1}]$

$$(\text{Planck's Constant}) = \frac{E}{u} = \frac{[M][LT^{-1}]^2}{T^{-1}} = ML^2T^{-1}$$

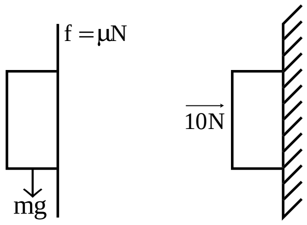
25. According to triangle law of forces, the resultant force is zero.

In presence of zero external force, there is no change in velocity

26. According to Gauss's Law

$\int (E.dA) = q_0/\varepsilon_0 \quad \Rightarrow q = \varepsilon_0(\phi_2 - \phi_1)$

$[\text{since}\, \phi = \int E.dA]$

27. 

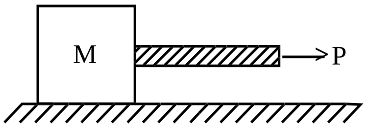
$f = mg \Rightarrow \mu N = W \Rightarrow \mu.10 = W$

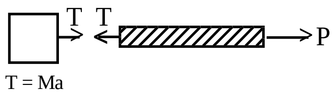
$\Rightarrow 0.2 \times 10 = W$ $\therefore W = 2N$

28. $a = \mu g = \frac{6}{10}$ [using v = u + at]

$\Rightarrow \mu = \frac{6}{10 \times g} = \frac{6}{10 \times 10} = 0.06$

31. Since the displacement for both block and rope is same so, the acceleration must be same for both





.... (i)

$\Rightarrow p = (m + M)a \qquad \Rightarrow a = \frac{P}{m+M}$

$T = M.a = \frac{PM}{m+M}$

33. Elastic energy = $\frac{1}{2} \times F \times x$

F = 200 N, x = 1 mm = 10^{-3} m $\therefore E = \frac{1}{2} \times 200 \times 1 \times 10^{-3} = 0.1J$

34. Escape velocity of a body is independent of the angle of projection. Hence, changing the angle of projection is not going to effect the magnitude of escape velocity

35. $T = 2\pi\sqrt{\frac{M}{K}}$ .... (i)

$\frac{5T}{3} = 2\pi\sqrt{\frac{M+m}{K}}$ .... (ii)

Dividing equation (ii) by equation (i), $\frac{5}{3} = \sqrt{\frac{M+m}{M}}$. Squaring both the sides

$\frac{25}{9} = \frac{M+m}{M} = 1 + \frac{m}{M} \Rightarrow \frac{m}{M} = \frac{25}{9} - 1 = \frac{16}{9}$

36. External amount of work must be done in order to flow heat from lower temperature to higher temperature. This is according to second law of thermodynamics.

37. $V_{max} = \omega A = m\omega^2 I = k$

$\Rightarrow \omega^2 = \frac{k}{m} \Rightarrow \omega = \sqrt{\frac{k}{m}}$ $\Rightarrow \omega \propto \sqrt{K}$ or $\frac{\omega_1}{\omega_2} = \sqrt{\frac{k_1}{k_2}}$

$V_A \max = V_B \max$ $\Rightarrow \left(\sqrt{\frac{k_1}{m}}\right)(A_1) = \left(\sqrt{\frac{k_2}{m}}\right)(A_2) \Rightarrow \frac{A_1}{A_2} = \sqrt{\frac{k_2}{k_1}}$

38. $T = 2\pi\sqrt{\frac{l}{g}}$; $\log T = \log(2\pi) + \frac{1}{2}\log\left(\frac{l}{g}\right)$ $\Rightarrow \log T = \log(2\pi) + \frac{1}{2}\log(l) - \frac{1}{2}\log(g)$

Differentiating

$\frac{\Delta T}{T} = 0 + \frac{1}{2} \times \frac{\Delta l}{l} - 0 \Rightarrow \frac{\Delta T}{T} \times 100 = \frac{1}{2} \times \frac{\Delta l}{l} \times 100$ $= \frac{1}{2} \times 21 = 10.5 \approx 10\%$

Note: In this method, the % error obtained is an approximate value on the higher side. Exact value is less than the obtained one.

39. $y = 10^{-4} \sin\left(600t - 2x + \frac{\pi}{3}\right)$. Comparing it with standard equation

$y = A\sin(vt - kx);$ $v = 600 m/s$

40. $e = -L\frac{dI}{dt} \Rightarrow 8 = (L)\frac{2-(-2)}{0.05} \Rightarrow L = 0.1H$

41. $q = \frac{Q}{\sqrt{2}}$

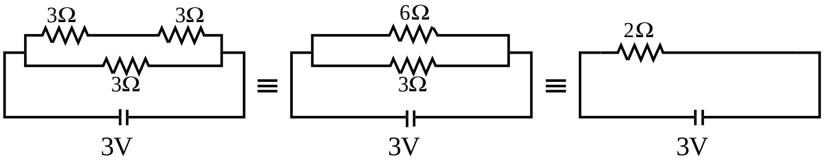
*44. $K = \frac{1}{f}\ln\left(\frac{N}{N}\right) \Rightarrow K = \frac{1}{5}\ln\left(\frac{5000}{1250}\right)$

$\frac{1}{5}\ln(4) = \frac{2}{5}\ln 2 = 0.4\ln 2$

45. No. of α particles emitted = 8, No. of β^- particles emitted = 4, No.of β^+ particles emitted = 2

z = 92 - 2 $\times$ 8 + 4 - 2 = 78

48. $I = \frac{3}{2} = 1.5A$



50. $x = 4(\cos\pi t + \sin\pi t) = 4[\sin\left(\frac{\pi}{2} - \pi t\right) + \sin\pi t] = 4\left[2 \times \sin\left(\frac{\pi t - \frac{\pi}{2} - \pi t}{2}\right)\cos\left(\frac{\pi t - \frac{\pi}{2} + \pi t}{2}\right)\right]$

$$= 8\left[\sin\frac{\pi}{4}.\cos\left(-\frac{\pi}{4}+\pi t\right)\right]$$

$$= \frac{8}{\sqrt{2}}.\cos\left[\pi t - \frac{\pi}{4}\right] = 4\sqrt{2}\cos\left[\pi t - \frac{\pi}{4}\right]$$

Comparing it with standard equation

X = A cos (wt - Kx) $\Rightarrow A = 4\sqrt{2}$

51. Potential due to spherical shell, $v_1 = \frac{q}{4\pi\varepsilon_0 R}$. Potential difference due to charge at the centre

$$V_2 = \frac{2Q}{4\pi\varepsilon_0 r}; V = V_1 + V_2 = \frac{2Q}{4\pi\varepsilon_0 R} + \frac{q}{4\pi\varepsilon_0 R}$$

52. Work done $= \frac{1}{2}\frac{q^2}{c} = \frac{(8\times10^{-18})^2}{2\times100\times10^{-8}} = 32\times10^{-32}$ J

53. $V_x = \frac{dx}{dt} = 3\alpha t^2,\ V_y = \frac{dy}{dt} = 3\beta t^2$

$$\vec{v} = \sqrt{V_{x^2} + V_{y^2}} = 3t^2\sqrt{\alpha^2 + \beta^2}$$

54. $P \propto T^3$ $\left(\frac{P_1}{P_2}\right) = \left(\frac{T_1}{T_2}\right)^3$

Comparing it with standard eq. $\frac{C_p}{C_v} = \gamma = \frac{3}{2}$

56. $\eta = \frac{(627+273)-(273+27)}{627+273}$

$$= \frac{900-300}{900} = \frac{600}{900} = \frac{2}{3}$$

work = $(\eta)\times$ Heat

$$= \frac{2}{3}\times3\times10^6\times4.2\,\text{J} = 8.4\times10^6\ \text{J}$$

57. Required work done

$$= \frac{1}{2}K(x_2^2 - x_1^2) = \frac{1}{2}\times5\times10^3[10^2 - 5^2]\times10^{-4}$$

$$= \frac{1}{2}\times5\times75\times10^3\times10^{-4} \quad = 18.75$$

58. $n = \frac{1}{2l}\sqrt{\frac{T}{\mu}};\ \ l = 1m$

T = 10 Kg wt. = 10 $\times$ 10 = 100 N

μ = 9.8 g/m = 9.8 $\times 10^{-3}$ kg/m n = 50 hz

66. Power = F. V

$$F = m\left(\frac{dV}{dt}\right) \Rightarrow m.v.\frac{dV}{dt} = \text{constant} = C$$

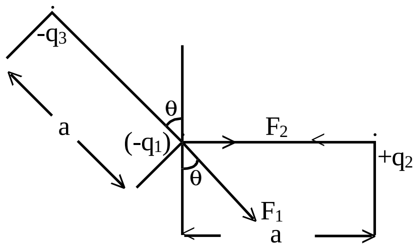
$$\Rightarrow \frac{dV}{dt} = \frac{C}{m} = k \Rightarrow vdv = kdt \qquad \Rightarrow \int v\,dv = \int k\,dt \Rightarrow \frac{V^2}{2} = kt + c$$

$$\Rightarrow v \propto (t)^{1/2} \qquad \frac{ds}{dt} = c.t^{1/2}$$

$$\Rightarrow \int ds = \int (c.t^{1/2})dt \Rightarrow S = C.\frac{2}{3}t^{3/2} \qquad \Rightarrow S = \frac{c.t^{3/2}}{3/2} \Rightarrow s \propto t^{3/2}$$

67. Thrust = Mass $\times$ Acceleration = $3.5 \times 10^4 \times 10 = 3.5 \times 10^5$ N

69. The force body diagram



$$F_1 = \frac{1}{4\pi\varepsilon_0}\cdot\frac{q_1q_3}{a^2}; \qquad F_2 = \frac{1}{4\pi\varepsilon_0}\cdot\frac{q_1q_3}{b^2}$$

$$F_X = F_1\sin\theta + F_2 = \frac{q_1}{4\pi\varepsilon_0}\left[\frac{q_3}{a^2}\sin\theta + \frac{q_1}{b_2}\right] \Rightarrow F_x \propto \left(\frac{q_3}{a^2}\sin\theta + \frac{q_2}{b^2}\right)$$

70. $p = \frac{V^2}{R}$ or $R = \frac{(220)^2}{1000}$

Power consumed = $\frac{V^2}{R} = \frac{110\times110}{220\times220}\times1000$ = 250 watt

73. According to Image formula

$$n = \frac{360}{\theta} - 1 \Rightarrow 3 = \frac{360}{\theta} - 1$$

$$\Rightarrow \frac{360}{\theta} = 4 \Rightarrow \theta = \frac{360}{4} = 90$$

74. $\frac{dH}{dt} \propto (\theta_2 - \theta_1) = (\Delta\theta)^n \Rightarrow n = 1$

75. $L_1 = 2l$ or $(\pi r^2 l) = (\pi r_2^2)(2l)$

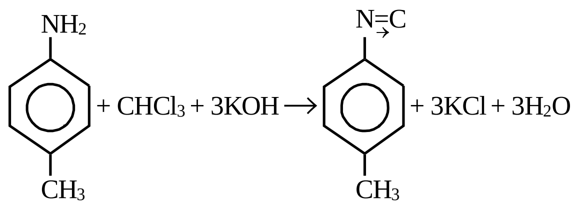
$$\Rightarrow r_2 = \frac{r}{\sqrt{2}}; \qquad R = \rho\frac{l}{\pi r^2}$$

$$R_{new} = (\rho)\frac{2l}{(\pi)(r/\sqrt{2})^2} = \frac{(\rho)4l}{(\pi)r^2} = 4\times R$$

$$\therefore \Delta R = 4R - R = 3R$$

$$\frac{\Delta R}{R}\% = \frac{3R}{R}\times100 = 300\%$$

76. Liquid hydrogen and liquid oxygen are used as excellent fuel for rockets. $H_2(l)$ has low mass and high enthalpy of combustion whereas oxygen is a strong supporter of combustion

77. 

78. Nylon is a polyamide polyer

79. More is the no . of + I groups attached to N atom greater is the basic character.

80. C_6H_5I will not respond to silver nitrate test because C-I bond has a partial double bond character.

81. For a cyclic process the net change in the internal energy is zero because the change in internal energy does not depend on the path.

82. $CH_2 = CH_2\ (g) + H_2(g) \rightarrow CH_3 - CH_3$

$\Delta H = 1(C=C)+4(C-H)+1(H-H)-1(C-C)-6(C-H) = 1(C=C)+1(H-H)-1(C-C)-2(C-H)$

$= 615+435-347-2\times 414 = 1050-1175 = -125\,kJ.$

83. ${}^{234}_{90}Th \xrightarrow{-\beta} {}^{234}_{91}X \xrightarrow{-\beta} {}^{234}_{92}Th \xrightarrow{-\alpha} {}^{230}_{90}Th$

84. $t_{1/2}$ = 3 hrs. Initial mass (C_0) = 256 g

$\therefore C_n = \frac{C_0}{2^n} = \frac{256}{(2)^6} = \frac{256}{64} = 4g$

86. $\Omega \propto \frac{1}{z}$

$\frac{\Omega_1}{\Omega_2} = \frac{z_2}{z_1} \Rightarrow \frac{1.06}{\Omega_2} = \frac{71}{57} \Rightarrow \Omega_2 = 0.85Å$

88. $Co(NH_3)_5\ Cl_3 [Co(NH_3)_5Cl]^{+2} + 2Cl^-$

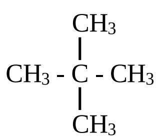
$\therefore$ Structure is $[Co(NH_3)_5\ Cl]\ Cl_2$.

89. $4\ (+1) + x + (-1) \times 4 = 0 \qquad \Rightarrow 4 + x - 4 = 0$

$x = 0$

91. An acidic solution cannot have a pH > 7.

92. In neopentane all the H atoms are same (1^0).



94. $PH_3 + 4Cl_2 \rightarrow PCl_5 + 3HCI$

95. $Fe^{+2} = 3d^6\ .\ 4s^0$

96. $4HCI + O_2 \rightarrow 2Cl_2 + 2H_2O$

Cloud of white fumes

99. The properties of elements change with a change in atomic number.

100. Ammonia can dissolve ppt. of Agcl only due to formation of complex as given below:

$AgCl + 2NH_3 \rightarrow [Ag(NH_3)_2]\ Cl$

101. Glass is a transparent or translucent super cooled liquid.

102. For s-electron, $l = 0$ $\therefore$ angular momentum = zero

103. Number of formulas in cube shaped crystals = $\frac{1.0}{58.5} \times 6.02 \times 10^{23}$ since in NaCl type of structure 4 formula units form a cell

$\therefore$ unit cells = $\frac{1.0 \times 6.02 \times 10^{23}}{58.5 \times 4}$ = 2.57×10^{21} unit cells.

104. $\mathrm{H-\overset{\overset{O}{\|}}{C}-O} \leftrightarrow \mathrm{H-\overset{\overset{O^{\ominus}}{|}}{C}=O}$

105. As adsorption is an exothermic process.

$\therefore$ Rise in temperature will decrease adsorption.

106. The equilibrium constant is related to the standard emf of cell by the expression

$$\log K = E^0_{cell} \times \frac{n}{0.059} = 0.295 \times \frac{2}{0.059}$$

$$\log K = \frac{590}{59} = 10 \text{ or } K = 1 \times 10^{-10}$$

107. For spontaneous reaction, $dS > 0$ an ΔG and dG should be negative i.e. < 0

108. $[A] = 1.0 \times 10^{-5}$, $[B] = [1.0 \times 10^{-5}]$

$K_{sp} = [2.B]^2 [A] = [2 \times 10^{-5}]^2 [1.0 \times 10^{-5}] = 4 \times 10^{-15}$

109. No. of moles of boron = $\frac{21.6}{10.8} = 2$

for BCl_3

$\therefore$ 1 mole of Boron = 3 mole of C*l*

$\therefore$ 2 mole of Boron = 6 mole of C*l*

$H_2 + Cl_2 \rightarrow 2HCl$

$\Rightarrow$ 3 moles of Hydrogen is required

$= 3 \times 22.4 = 67.2$ Litre

110. $K_C = \frac{[NO_2]^2}{[N_2O_4]} = \frac{[1.2 \times 10^{-2}]^2}{[4.8 \times 10^{-2}]} = 3 \times 10^{-3}$ mol/L

111. Due to exothermicity of reaction low or optimum temperature will be required. Since 3 moles are changing to 2 moles.

$\therefore$ High pressure will be required.

113. $\underset{\text{(insoluble)}}{HgI_2} + KI \longrightarrow \underset{\text{(soluble)}}{K_2HgI_4}$

On heating HgI_2 decomposes as $HgI_2 \rightleftharpoons Hg + I_2$

117. No. of moles of silver = $\frac{9650}{96500} = \frac{1}{10}$ moles

$\therefore$ Mass of silver deposited = $\frac{1}{10} \times 108 = 10.8\,g$

118. $E_{cell} = E^0_{cell} + \frac{0.059}{n}\log\frac{[Cu^{+2}]}{[Zn^{+2}]}$

$= 1.10 + \frac{0.059}{2}\log[0.1] = 1.10 - 0.0295 = 1.07\,V$

120. f-block elements show a regular decrease in atomic size due to lanthanide/actinide contraction.

122. $LiAl^{H}_{4}$ can reduce COOH group and not the double bond

$CH_2 = CH - COOH \xrightarrow{LAH} CH_2{=}CH{-}CH_2OH$

123. According to kinetic theory the gas molecules travel in a straight line path but show haphazard motion due to collisions.

125. A chiral object or structure has four different groups attached to the carbocation.

126. $Cr_2O_7^{2-} + OH^- \rightarrow 2CrO_4^{2-} + H^+$. The above equilibrium shifts to L.H.S. on addition of acid.

127. It is because mercury exists as liquid at room temperature.

128. Gypsum is $CaSO_4.2H_2O$

129. $(C_6H_{10}O_5)n + nH_2O \xrightarrow{H^+} \underset{D-Glucose}{nC_6H_{12}O_6}$

130. $AgNO_3 \rightarrow Ag + NO_2 + \frac{1}{2}O_2$

131. $CH_3CH_2OH + H^+ \xrightarrow{step\ 1} CH_3CH_2-\overset{\oplus}{O}(H)-H$

Protonated alcohol

132. The solubility is governed by $\Delta H_{solution}$ i.e. $\Delta H_{solution} = \Delta H_{lattice} - \Delta H_{Hydration}$.

Due to increase in size the magnitude of hydration energy decresaes and hence the solubility.

133. The rain water after thunderstorm contains dissolved acid and therefore the pH is less than rain water without thunderstorm.

135. $Ba(OH)_2 + 2HCI \rightarrow BaCl_2 + 2H_2O$

Applying Molarity equation, $\frac{M_1V_1}{\underset{(Ba(OH)_2)}{1}} = \frac{M_2V_2}{\underset{(HCI)}{2}}$ or $25 \times M_1 = \frac{0.1 \times 35}{2}$ $\therefore M_1 = \frac{0.1\times35}{2\times25} = \frac{0.7}{10} = 0.07$

137. $Rate_1 = k\,[A]^n\,[B]^m$; $Rate_2 = k[2A]^n\,[½B]^m$

$\therefore \frac{Rate_2}{Rate_1} = \frac{k[2A]^n\,[½B]^m}{k[A]^n\,[B]^m} = [2]^n\,[½]^m = 2^n.2^{-m} = 2^{n-m}$

138. $CH_3CH_2N \rightleftarrows C + H_2O \xrightarrow{H^+} CH_3CH_2NH_2$ + HCOOH. Therefore it gives only one mono chloroalkane.

140. On increasing pressure, the temperature is also increased. Thus in pressure cooker due to increase in pressure the b.p. of water increases.

141. $r = k[O_2]\,[NO]^2$. When the volume is reduced to 1/2, The conc. will double.

$\therefore$ New rate $= k[2O_2]\,[2NO]^2 = 8\,k\,[O^2]\,[NO]^2$. The new rate increases by eight times.

142. Magnesium provides cathodic protection and prevent rusting or corrosion.

143. Both NO_2 and O_3 have angular shape and hence will have net dipole moment.

144. N^{3-}, F^- and Na^+ contain 10 electrons each.

145. Permanent hardness of water is due to chlorides and sulphates of calciumand magnesium.

146. In H_2S, due to low electronegativity of sulphur the L.P. - L.P. repulsion is more than B.P. - B.P. repulsion and hence the bond angle is 92^0.

147. Both XeF_2 and and CO_2 have a linear structure.

148. Electronic configuration of Cr is

3d						4s
↑	↑	↑	↑	↑		↑

So due to half filled orbital I.P. is high of Cr.

149. The lines falling in the visible region comprise Balmer series. Hence the third line would be $n_1 = 2$, $n_2 = 5$ i.e. $5 \rightarrow 2$.

150. $\lambda = \frac{h}{mv} = \frac{6.6\times10^{-34}}{60\times10^{-3}\times10} = 10^{-33} m$

1. $\frac{d}{dx}F(x) = \frac{e^{\sin x}}{x}$ or $\int_1^4 \frac{3}{x} e^{\sin x^3} dx = \int_1^4 \frac{3x^2}{x^3} e^{\sin x^3} dx$

Let $x^3 = t$, $3x^2dx = dt$

when x = 1, t = 1 & x - 4, t = 64

$$F(t) = \int_1^{64} \frac{e^{\sin t}}{t} dt = \int_1^{64} F(t)\,dt = F(64) - F(1)$$

K = 64.

2. n = 9 then median term = $\left(\frac{9+1}{2}\right)^{th}$ = 5^{th} term. Last four observations are increased by 2. The median is 5th observation which is remaining unchanged. $\therefore$ There will be no change in median.

3. $\lim_{n\to\infty}\left\{\left(\frac{1}{n}\right)^4 + \left(\frac{2}{n}\right)^4 + \left(\frac{3}{n}\right)^4 + \ldots\ldots\ldots\ldots \left(\frac{n}{n}\right)^4\right\} - \lim_{n\to\infty}\frac{1}{n}\left\{\frac{1}{n^4} + \frac{2^3}{n^4} + \ldots\ldots\ldots \frac{n^3}{n^4}\right\}$

$$\int_0^1 (x)^4 dx - 0 = \left[\frac{x^5}{5}\right]_0^1 = \frac{1}{5}$$

4. Fundamental theorem (fact) $t_2 = -t_1 - \frac{2}{t_1}$

5. $|r_1 - r_2| = C_1C_2$ for intersection

$\Rightarrow$ r - 3 < 5 $\Rightarrow$ r < 8 .... (1)

and $r_1 + r_2 > C_1C_2$, r+3 > 5 $\Rightarrow$ r = 2 ....(2)

From (1) and (2), 2 < r < 8.

6. $y^2 = 4a(x - h)$, $2yy_1 = 4a \Rightarrow yy_1 = 2a \Rightarrow y_1^2 + yy_1 = 0$

Degree = 1, order = 2,

7. $\frac{x^2}{144} - \frac{y^2}{81} = \frac{1}{25}$

$$a = \sqrt{\frac{144}{25}},\ b = \sqrt{\frac{81}{25}},\ e = \sqrt{1 + \frac{81}{144}} = \frac{15}{12} = \frac{5}{4}$$

Foci = (3, 0), focus of ellipse = (3, 0) $\Rightarrow e = \frac{3}{4}$

$$b^2 = 16\left(1 - \frac{9}{16}\right) = 7$$

8. $F(t) = \int_0^t f(t-y)g(y)\,dy$

$$= \int_0^t e^{t-y} y\,dy = e^t \int_0^t e^{-y} y\,dy$$

$$= e^t \left[-ye^{-y} - e^{-y}\right]_0^t = -e^t \left[ye^{-y} + e^{-y}\right]_0^t$$

$$= -e^t \left[te^{-t} + e^{-t} - 0 - 1\right] = e^t \left[\frac{t+1-e^t}{e^t}\right] = e^t - (1+t)$$

9. $f(x) = \log\left(x + \sqrt{x^2+1}\right)$

$f(-x) = -\log\left(x + \sqrt{x^2+1}\right)$

$f(-x) = -f(x)$, i.e., $f(x)$ is an odd function.

10. $ax^2 + bx + c = 0,\ \alpha + \beta = \frac{-b}{a},\ \alpha\beta = \frac{c}{a}$

As for given condition, $\alpha + \beta = \frac{1}{\alpha^2} + \frac{1}{\beta^2}$

$$\alpha + \beta = -\frac{\alpha^2 + \beta^2}{\alpha^2 \beta^2} - \frac{b}{a} = \frac{\frac{b^2}{a^2} - \frac{2c}{a}}{\frac{c^2}{a^2}}$$

On simplification $2a^2c = ab^2 + bc^2$

$\Rightarrow \frac{2a}{b} = \frac{c}{a} + \frac{b}{c}$ $\quad \therefore \frac{a}{b}, \frac{b}{a}, \& \frac{c}{b}$ are in H.P.

11. $\begin{vmatrix} 1 & 2a & a \\ 1 & 3b & b \\ 1 & 4c & c \end{vmatrix} = 0 \qquad C_2 \to C_2 - 2C_3$

$\begin{vmatrix} 1 & 0 & a \\ 1 & b & b \\ 1 & 2c & c \end{vmatrix} = 0 \qquad R_3 \to R_3 - R_2,\ R_2 \to R_2 - R_1$

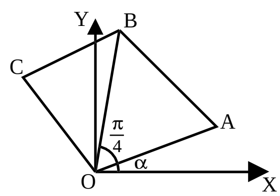
$\begin{vmatrix} 1 & 0 & a \\ 0 & b & b-a \\ 0 & 2c-b & c-b \end{vmatrix} = 0$

b (c - b) - (b - a) (2c - b) = 0

On simplification, $\frac{2}{b} = \frac{1}{a} + \frac{1}{c}$

$\therefore$ a, b, c are in Harmonic Progression.

12. Co-ordinates of A = $(a\cos\alpha, a\sin\alpha)$



Equation of OB, $y = \tan\left(\frac{\pi}{4} + \alpha\right)x$

CA $\perp$ r to OB $\quad\therefore$ slope of CA $= -\cot\left(\frac{\pi}{4}+2\right)$

Equation of CA $y - a\sin\alpha = -\cot\left(\frac{\pi}{4}+2\right)(x - a\cos\alpha)$.

$y(\sin\alpha + \cos\alpha) + x(\cos\alpha - \sin\alpha) = a$

13. Equation of bisector of both pair of straight lines,

$px^2 + 2xy - py^2 = 0$ ... (1)

$qx^2 + 2xy - qy^2 = 0$ .... (2)

From (1) and (2). $\frac{q}{1} = \frac{2}{-2p} = \frac{-q}{-1} \Rightarrow pq = -1$.

14. $x = \frac{\cos t + b\sin t + 1}{3} \Rightarrow a\cos t + b\sin t = 3x - 1$

$y = \frac{a\sin t - b\cos t}{3} \Rightarrow a\sin t - b\sin t = 3y$

Squaring & adding, $(3x - 1)^2 + (3y)^2 = a^2 + b^2$

15. $\lim_{x\to 0} \frac{\log(3+x) - \log(3-x)}{x} = K$ (by L'Hospital rule)

$\lim_{x\to 0} \frac{\frac{1}{3+x} - \frac{-1}{3-x}}{1} = K \quad \therefore \frac{2}{3} = K$

16. $\vec{a} = \vec{r}\times\vec{p};\ |\vec{a}| = rp\sin\theta$

$|\vec{H}| = rp\cos\theta \quad \left[\because \sin(90^0 + \theta) = \cos\theta\right]$

G = rp sin θ .... (1)

H = rp cos θ .... (2)

x = rp sin $(\theta + \alpha)$ .... (3)

From (1), (2) & (3),

$x = \vec{a}\cos\alpha + \vec{H}\sin\alpha$

17. $R^2 = P^2 + Q^2 + 2PQ\cos\theta$ .... (1)

$4R^2 = P^2 + 4Q^2 + 4PQ\cos\theta$ .... (2)

$4R^2 = P^2 + Q^2 - 2PQ\cos\theta$ .... (3)

On (1) + (2), $5R^2 = 2P^2 + 2Q^2$ .... (4)

On (3)$\times$2+(2), $12R^2 = 3P^2 + 6Q^2$ .... (5)

$2P^2 + 2Q^2 - 5R^2 = 0$ .... (6)

$3P^2 + 6Q^2 - 12R^2 = 0$ .... (7)

$$\frac{P^2}{-24+30} = \frac{Q^2}{24-15} = \frac{R^2}{12-6}$$

$\frac{P^2}{6} = \frac{Q^2}{9} = \frac{R^2}{6}$ or $P^2 : Q^2 : R^2 = 2:3:2$

18. $\left.\begin{matrix} np = 4 \\ npq = 2 \end{matrix}\right\} \Rightarrow q = \frac{1}{2}, p = \frac{1}{2}, n = 8$

$$p(X=1) = {}^{8}C_1\left(\frac{1}{2}\right)\left(\frac{1}{2}\right)^7 = 8.\frac{1}{2^8} = \frac{1}{2^5} = \frac{1}{32}$$

19. $f(x) = x^n \Rightarrow f(1) = 1$

$f'(x) = nx^{n-1} \Rightarrow f'(1) = n$

$f''(x) = n(n-1)x^{n-2} \Rightarrow f''(1) = n(n-1)$

.... $f^n(x) = n! \Rightarrow f^n(1) = n!$

$$= 1 - \frac{n}{1!} + \frac{n(n-1)}{2!} - \frac{n(n-1)(n-2)}{3!} + ... + (-1)^n \frac{n!}{n!}$$

$$= {}^nC_0 - {}^nC_1 + {}^nC_2 - {}^nC_3 + ... + (-1)^n\, {}^nC_n = 0$$

20. Since $\vec{n}$ is perpendicular $\vec{u}$ and $\vec{v}$, $\vec{n} = \vec{u} \times \vec{v}$

$$\hat{n} = \frac{\begin{vmatrix} i & j & k \\ 1 & 1 & 0 \\ 1 & -1 & 0 \end{vmatrix}}{\sqrt{2}\times\sqrt{2}} = \frac{-2\hat{k}}{2} = -\hat{k}$$

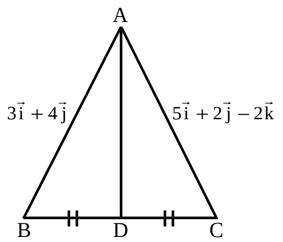
$$|\vec{\omega}.\hat{n}| = |(i+2j+3k).(-k)| = |-3| = 3$$

21. $\vec{F} + \vec{F}_1 + \vec{F}_2 = 7i + 2j - 4k$

$\vec{d} = \text{P.V of } \vec{B} - \text{P.V of } \vec{A} = 4i + 2j - 2k$

$W = \vec{F}.\vec{d} = 28 + 4 + 8 = 40$ unit

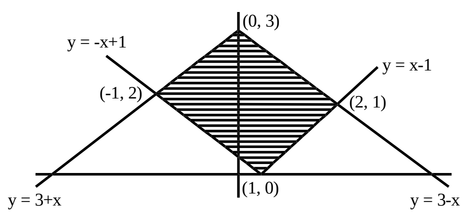
22.



$$\text{P.V of } \overrightarrow{AD} = \frac{(3+5)i + (0-2)j + (4+4)k}{2}$$

$$= 4i - j + 4k \text{ or } |\overrightarrow{AD}| = \sqrt{16+16+1} = \sqrt{33}$$

23.



$$A = \int_{-1}^{0}\{(3+x)-(-x+1)\}dx + \int_{0}^{1}\{(3-x)-(-x+1)\}dx + \int_{1}^{2}\{(3-x)-(-x-1)\}dx$$

$$= \int_{-1}^{0}(2+2x)dx + \int_{0}^{1}2dx + \int_{1}^{2}(4-2x)dx$$

$$= \left[2x - x^2\right]_{-1}^{0} + \left[2x\right]_0^1 + \left[4x - x^2\right]_1^2$$

= 0 - (-2+1) + (2 - 0) + (8 - 4) - (4 - 1)

= 1 + 2 + 4 - 3 = 4 sq. units

24. Shortest distance = perpendicular distance $= \left|\frac{-2\times12+4\times1+3\times3-327}{\sqrt{144+9+16}}\right| = 26$

$\therefore$ Shortest distance

$= 26 - \sqrt{4+1+15+9} = 26 - 13 = 13 \quad [\because 26 - r]$

25. $\frac{x-b}{a} = \frac{y}{1} = \frac{3-d}{c}; \frac{x-b'}{a'} = \frac{y}{1} = \frac{3-d'}{c'}$

For perpendicular $aa' + 1 + cc' = 0$

26. $\begin{vmatrix} x_2 - x_1 & y_2 - y_1 & z_2 - z_1 \\ l_1 & m_1 & n_1 \\ l_2 & m_2 & n_2 \end{vmatrix} = 0$

$\begin{vmatrix} 1 & -1 & -1 \\ 1 & 1 & -k \\ k & 2 & 1 \end{vmatrix} = 0 \Rightarrow \begin{vmatrix} 0 & 0 & -1 \\ 2 & 1+k & -k \\ k+2 & 1 & 1 \end{vmatrix} = 0$

$k^2 + 3k^2 = 0 \Rightarrow k(k + 3) = 0$ or $k = 0$ or -3

27. $I = \int_a^b xf(x)\,dx = \int_a^b (a+b-x)f(a+b-x)\,dx$

$$= (a+b)\int_a^b f(a+b-x)\,dx - \int_a^b xf(a+b-x)\,dx$$

$$= (a+b)\int_a^b f(a+b-x)\,dx - \int_a^b xf(x)\,dx$$

$$2I = (a+b)\int_a^b f(x)\,dx$$

$$I = \frac{(a+b)}{2}\int_a^b f(x)\,dx;\ I = \frac{(a+b)}{2}\int_a^b f(a+b-x)\,dx$$

28. Portion OA, OB corresponds to motion with acceleration ‘f’ and retardation ‘r’ respectively.

Area of $\Delta OAB = S$ and $OB = t$. Let $OL = t_1$,

$LB = t_2$ and $AL = v$, $S = \frac{1}{2} OB.AL = \frac{1}{2} t.v;\ v = \frac{2S}{t}$

Also, $f = \frac{v}{t_1},\ t_1 = \frac{v}{f} = \frac{2s}{tf}$ and $r = \frac{v}{t_2}, t_2 = \frac{v}{r} = \frac{2s}{tr}; t = t_1 + t_2 = \frac{2s}{tf} + \frac{2s}{tr}$

$t = \left(\frac{1}{f} + \frac{1}{r}\right)\frac{2s}{t} \quad \Rightarrow t = \sqrt{2s\left(\frac{1}{f} + \frac{1}{r}\right)}$

29. $R = u\sqrt{\frac{2h}{g}} = (u\cos\theta)\times t$

$t = \frac{1}{\cos\theta}\sqrt{\frac{2h}{g}}$ .... (1)

Now, $h = (-u\sin\theta)t + \frac{1}{2}gt^2$

Substituting 't' from (1),

$h = -\frac{u\sin\theta}{\cos\theta}\sqrt{\frac{2h}{g}} + \frac{1}{2}g\left[\frac{2h}{g\cos^2\theta}\right]$ $\qquad h = -u\sqrt{\frac{2h}{g}}\tan\theta + h\sec^2\theta$

$h = -u\sqrt{\frac{2h}{g}}\tan\theta + h\tan^2\theta + h$

$\tan^2\theta - u\sqrt{\frac{2}{hg}}\tan\theta = 0; \quad \therefore \tan\theta = u\sqrt{\frac{2}{hg}}$

30. Applying $R_1 \to R_1 + R_2 + R_3$

As, $1 + \omega^n + \omega^{2n} = 0; \quad \therefore \Delta = 0$

31. $\tan\left(\frac{\pi}{n}\right) = \frac{a}{2r};\ \sin\left(\frac{\pi}{n}\right) = \frac{a}{2R}$

$r + R = \frac{a}{2}\left[\cot\frac{\pi}{n} + \operatorname{cosec}\frac{\pi}{n}\right] \Rightarrow r + R = \frac{a}{2}.\cot\left(\frac{\pi}{2n}\right)$

32. Taking co-ordinates as $\left(\frac{x}{r}, \frac{y}{r}\right); (x, y) \& (xr, yr)$. Above coordinates satisfy the relation $y = mx$ Therefore lies on the straight line.

33. $|z\omega| = 1$ ..... (1)

As, $\operatorname{Arg}\left(\frac{z}{\omega}\right) = \frac{\pi}{2}$ therefore $\frac{z}{\omega} = i$

$\therefore \left|\frac{z}{\omega}\right| = 1$ .... (2)

From (1) & (2), $|z| = |\omega| = 1$ and $\frac{z}{\omega} + \frac{\bar{z}}{\omega} = 0; z\bar{\omega} + \bar{z}\omega = 0$

$\bar{z}\omega = -z\bar{\omega} = \frac{-z}{\omega}.\bar{\omega}.\omega;\ \bar{z}\omega = -i|\omega|^2 = -i$

34. $z^2 + az + b = 0;\ z_1 + z_2 = -a\ \&\ z_1z_2 = b$

$0, z, z_2$ form an equilateral Δ

$\therefore\ 0^2 + z_1^2 + z_2^2 = 0.z_1 + z_1.z_2 + z_2.0$

(for equation Δ, $z_1^2 + z_2^2 + z_3^2 = z_1z_2 + z_2z_3 + z_3z_1$)

$z_1^2 + z_2^2 = z_1z_2$ or $(z_1 + z_2)^2 = 3z_1z_2$

$\therefore\ a^2 = 3b.$

35. $(1+y^2)+(x-e^{\tan^{-1}y})\dfrac{dy}{dx}=0$

$$(1+y^2)\frac{dx}{dy}+x=e^{\tan^{-1}y}\Rightarrow\frac{dx}{dy}+\frac{x}{(1+y^2)}=\frac{e^{\tan^{-1}y}}{(1+y^2)}$$

$$\text{I.F.} = e^{\int\frac{1}{(1+y^2)}dy} = e\tan^{-1}y$$

$$x(e^{\tan^{-1}y})=\int\frac{e^{\tan^{-1}y}}{1+y}\ e^{\tan^{-1}y}dy$$

$$x(e^{\tan^{-1}y})=\frac{e^{2\tan^{-1}y}}{2}+C\quad\therefore\ 2xe^{\tan^{-1}y}=e^{2\tan^{-1}y}+k$$

36. Let $f(x) = e^x$

$$\therefore\int_0^1 f(x)g(x)\,dx=\int_0^1 e^x(x^2-e^x)dx$$

$$=\int_0^1 x^2e^x dx-\int_0^1 e^{2x}dx$$

$$=\left[x^2e^x\right]_0^1-2\left[xe^x-e^x\right]_0^1-\frac{1}{2}\left[e^{2x}\right]_0^1$$

$$=e-\left[\frac{e^2}{2}-\frac{1}{2}\right]-2[e-e+1]=e-\frac{e^2}{2}-\frac{3}{2}$$

37. $\pi r^2 = 154 \Rightarrow r = 7$

For centre on solving equation

$2x - 3y = 5\ \&\ 3x - 4y = 7$ or $x = 1, y = 1$ centre = $(1, -1)$

Equation of circle, $(x - 1)^2 + (y + 1)^2 = 7^2$

$x^2 + y^2 - 2x + 2y = 47$

38. $P(A)=\dfrac{3x+1}{3},\ P(B)=\dfrac{1-x}{4},\ P(C)=\dfrac{1-2x}{2}$

These are mutually exclusive

$$0\le\frac{3x+1}{3}\le 1,\quad 0\le\frac{1-x}{4}\le 1 \text{ and } 0\le\frac{1-2x}{2}\le 1$$

$$-1\le 3x\le 2,\ -3\le x\le 1 \text{ and } -1\le 2x\le 1$$

$$-\frac{1}{3} \le x \le \frac{2}{3}, -3 \le x \le 1, \text{ and } -\frac{1}{2} \le x \le \frac{1}{2}$$

Also $0 \le \frac{1+3x}{3} + \frac{1-x}{4} + \frac{1-2x}{2} \le 1$

$$0 \le 13 - 3x \le 12 \Rightarrow 1 \le 3x \le 13 \Rightarrow \frac{1}{3} \le x \le \frac{13}{3}$$

$$\max\left\{-\frac{1}{3}, -3, -\frac{1}{2}, \frac{1}{3}\right\} \le x \le \min\left\{\frac{2}{3}, 1, \frac{1}{2}, \frac{13}{3}\right\}$$

$$\frac{1}{3} \le x \le \frac{1}{2} \Rightarrow x \in \left[\frac{1}{3}, \frac{1}{2}\right]$$

39. $n(S) = {}^5C_2$; $n(E) = {}^2C_1 + {}^2C_1$

$$p(E) = \frac{n(E)}{n(S)} = \frac{{}^2C_1 + {}^2C_1}{{}^5C_2} = \frac{2}{5}$$

40. $3\alpha = \frac{1-3a}{a^2 - 5a + 3}$ & $2\alpha^2 = \frac{2}{a^2 - 5a + 3}$

$$2\left[\frac{1}{9}\frac{(1-3a)^2}{(a^2-5a+3)^2}\right] = \frac{2}{a^2 - 5a + 3}$$

$$\frac{(1-3a)^2}{(a^2-5a+3)} = 9 \text{ or } 9a^2 - 6a + 1$$

$$= 9a^2 - 45a + 27 \quad \text{or} \quad 39a = 26 \quad \text{or } a = \frac{2}{3}$$

41. $T_{r+1} = \frac{n(n-1)(n-2)..........(n-r+1)}{r!}(x)^r$

For first negative term, n - r + 1 < 0 or $r > \frac{32}{5}$

$\therefore$ r = 7. Therefore, first negative term is T_8.

42. $T_{r+1} = {}^{256}C_r(\sqrt{3})^{256-r}(\sqrt[8]{5})^r = {}^{256}C_r(3)^{\frac{256-r}{2}}(5)^{r/8}$

Terms will be integral if $\frac{256-r}{2}$ & $\frac{r}{8}$ both are +ve integer. As $0 \le r \le 256 \;\therefore\; r = 0, 8, 16, 24, ...256$

For above values of $r, \left(\frac{256-r}{2}\right)$ is also an integer.

43. After t; velocity = $f \times t$

$V_{BA} = \vec{f}\,t + (-\vec{u}) = \sqrt{f^2 t^2 + u^2 - 2fut\cos\alpha}$

For max. and min.

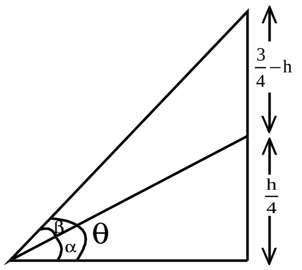
$$\frac{d}{dt}\left(V_{BA}^2\right) = 2f^2 t - 2fu\cos\alpha = 0 \quad \text{or} \quad t = \frac{u\cos\alpha}{f}$$

Therefore, total no. of values of r = 33.

44. Using ${}^nC_r + {}^nC_{r-1} = {}^{n+1}C_r = {}^nC_{r+1} + \underbrace{{}^nC_{r-1} + {}^nC_r + {}^nC_r} = {}^nC_{r+1} + {}^{n+1}C_r + {}^nC_r$

$${}^{n+1}C_{r+1} + {}^{n+1}C_r \Rightarrow {}^{n+2}C_{r+1}$$

45.



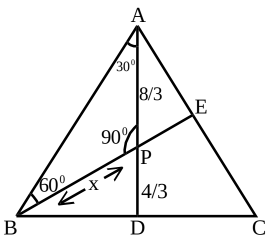
$\theta = \alpha + \beta$ or $\beta = \theta - \alpha$

$$\tan\beta = \frac{\tan\theta - \tan\alpha}{1 + \tan\theta . \tan\alpha} \quad \text{or} \quad \frac{3}{5} = \frac{\frac{h}{40} - \frac{h}{160}}{1 + \frac{h}{40} . \frac{h}{160}}$$

h^2 - 200 h + 6400 = 0, h = 40 or 160 metre

Therefore possible height = 40 metre

46.



$$\tan 60^0 = \frac{8/3}{x} \quad \text{or} \quad x = \frac{8}{3\sqrt{3}}$$

$$\text{Area of } \Delta ABD = \frac{1}{2} \times 4 \times \frac{8}{3\sqrt{3}} = \frac{16}{3\sqrt{3}} \qquad \therefore \text{Area of } \Delta ABC = 2 \times \frac{16}{3\sqrt{3}} = \frac{32}{3\sqrt{3}}$$

47. If $a\cos^2\left(\frac{C}{2}\right) + c\cos^2\left(\frac{A}{2}\right) = \frac{3b}{2}$

a[cos C + 1] + c[cos A + 1] = 3b

(a + c) + (a cos C + c cos B) = 3b

a + c + b = 3b or a + c = 2b or a, b, c are in A.P.

48. $\vec{a} + \vec{b} + \vec{c} = 0 \Rightarrow (\vec{a} + \vec{b} + \vec{c}).(\vec{a} + \vec{b} + \vec{c}) = 0$

$$|\vec{a}|^2 + |\vec{b}|^2 + |\vec{c}|^2 + 2(\vec{a}.\vec{b} + \vec{b}.\vec{c} + \vec{c}.\vec{a}) = 0$$

$$\vec{a}.\vec{b} + \vec{b}.\vec{c} + \vec{c}.\vec{a} = \frac{-1-4-9}{2} = -7$$

49. $I = \int_0^1 x(1-x)^n\, dx$

$$-I = \int_0^1 -x(1-x)^n\, dx = \int_0^1 (1-x-1)(1-x)^n\, dx$$

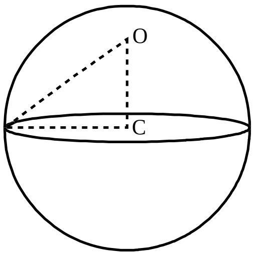
$$= \int_0^1 (1-x)^{n+1}\,dx - \int_0^1 (1-x)^n\,dx$$

$$= \left[\frac{(1-x)^{n+2}}{-(n+2)}\right]_0^1 - \left[\frac{(1-x)^{n+1}}{-(n+1)}\right]_0^1 = \frac{1}{n+2} - \frac{1}{n+1}$$

$$I = \frac{1}{n+1} - \frac{1}{n+2}$$

50. $$\lim_{x\to 0} \frac{\frac{d}{dx}\int_0^{x^2} \sec^2 t\,dt}{\frac{d}{dx}(x\sin x)} = \lim_{x\to 0} \frac{\sec^2 x^2 . 2x}{\sin x + x\cos x} \text{ (by L' Hospital rule)}$$

$$\lim_{x\to 0} \frac{2\sec^2 x^2}{\left(\frac{\sin x}{x} + \cos x\right)} = \frac{2\times 1}{1+1} = 1$$

51. 

centre of sphere = (-1, 1, 2)

Radius of sphere $\sqrt{1+1+4+19} = 5$

Perpendicular distance from centre to the plane

$$OC = d = \left|\frac{-1+2+4+7}{\sqrt{1+4+4}}\right| = \frac{12}{3} = 4.$$

$AC^2 = AO^2 - OC^2 = 5^2 - 4^2 = 9 \Rightarrow AC = 3$

52. Vector perpendicular to the face OAB

$$= \overrightarrow{OA} \times \overrightarrow{OB} = \begin{vmatrix} i & j & k \\ 1 & 2 & 1 \\ 2 & 1 & 3 \end{vmatrix} = 5i - j - 3k$$

Vector perpendicular to the face ABC

$$= \overrightarrow{AB} \times \overrightarrow{AC} = \begin{vmatrix} i & j & k \\ 1 & -1 & 2 \\ -2 & -1 & 1 \end{vmatrix} = i - 5j - 3k$$

Angle between the faces = Angle between their normals

$$\cos\theta = \left|\frac{5+5+9}{\sqrt{35}\sqrt{35}}\right| = \frac{19}{35} \text{ or } \theta = \cos^{-1}\left(\frac{19}{35}\right)$$

53. $\lim_{x\to a} \dfrac{k9(x)-kf(x)}{9(k)-f(x)} = 4$ (By L'Hospital rule)

$\lim_{x\to a} k\dfrac{9'(x)-f'(x)}{9'(x)-f'(x)} = 4$ or $k = 4$.

54. $\lim_{x\to \frac{\pi}{2}} \dfrac{\tan\left(\frac{\pi}{4}-\frac{x}{2}\right).(1-\sin x)}{(\pi-2x)^3}$

Let $x = \frac{\pi}{2}+y;\ y \to 0$

$= \lim_{y\to 0} \dfrac{-\tan\left(-\frac{y}{2}\right).(1-\cos y)}{(-2y)^3} = \lim_{y\to 0} \dfrac{-\tan\frac{y}{2}\,2\sin^2\frac{y}{2}}{(-8).\frac{y^3}{8}.8}$

$= \lim_{y\to 0} \dfrac{1}{32}\dfrac{\tan\frac{y}{2}}{\left(\frac{y}{2}\right)}.\left[\dfrac{\sin y/2}{y/2}\right]^2 = \dfrac{1}{32}$

55. $(h-a_1)^2+(k-b_1)^2=(h-a_2)^2+(k-b_2)^2$

$(a_1-a_2)x+(b_1-b_2)y+\frac{1}{2}(a_2^2+b_2^2-a_1^2-b_1^2)=0$

$C=\frac{1}{2}(a_2^2+b_2^2-a_1^2-b_1^2)$

56. $\begin{vmatrix} a & a^2 & 1+a^3 \\ b & b^2 & 1+b^3 \\ c & c^2 & 1+c^3 \end{vmatrix}=0 \Rightarrow \begin{vmatrix} a & a^2 & 1 \\ b & b^2 & 1 \\ c & c^2 & 1 \end{vmatrix}+\begin{vmatrix} a & a^2 & a^3 \\ b & b^2 & b^3 \\ c & c^2 & c^3 \end{vmatrix}=0$

(a - b) (b - c) (c - a) + abc(a - b) (b - c) (c - a) = 0

(abc + 1) [(a - b) (b - c) (c - a)] = 0

As $\begin{vmatrix} 1 & a & a^2 \\ 1 & b & b^2 \\ 1 & c & c^2 \end{vmatrix} \neq 0$ (given condition) $\therefore$ abc = -1

57. $x^2-3|x|+2=0$ or $(|x|-2)(|x|-1)=0$

$|x|=1,2$ or $x=\pm1,\pm2$ or $\therefore$ No. of solution $=4$

58. $f(x) = 2x^3 - 9ax^2 + 12a^2x + 1$

$f'(x) = 6x^2 - 18ax + 12a^2$; $f''(x) = 12x - 18a$

For max. or min. $6x^2 - 18ax + 12a^2 = 0 \Rightarrow x^2 - 3ax + 2a^2 = 0$

x = a or x = 2a, at x = a max.and at x = 2a min.

$p^2 = q$

$a^2 = 2a \Rightarrow a = 2$ or $a = 0$

but a > 0, therefore, a = 2.

59. $f(0)=0;\ f(x)=xe^{-\left(\frac{1}{|x|}+\frac{1}{x}\right)}$

R.H.L. $\lim_{h\to 0}(0+h)e^{-2/h}=\lim_{h\to 0}\frac{h}{e^{2/h}}=0$

L.H.L $\lim_{h\to 0}(0-h)e^{-\left(\frac{1}{h}-\frac{1}{h}\right)}=0$

Therefore, f(x) is continuous

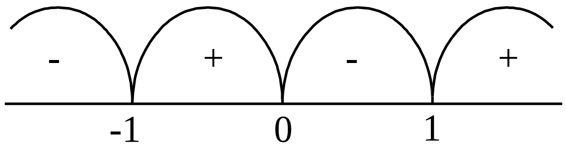
R.H.D. $\lim_{h\to 0}\frac{(0+h)e^{-\left(\frac{1}{h}+\frac{1}{h}\right)}-he^{-\left(\frac{1}{h}+\frac{1}{h}\right)}}{h}=0$

L.H.D. $\lim_{h\to 0}\frac{(0-h)e^{-\left(\frac{1}{h}-\frac{1}{h}\right)}-he^{-\left(\frac{1}{h}+\frac{1}{h}\right)}}{-h}=1$

Therefore, L.H.D. $\neq$ R.H.D.

f(x) is not differentiable at x = 0.

60.



$f(x)=\frac{3}{4-x^2}+\log_{10}(x^3-x)$

$4-x^2\neq 0; x^3-x>0; x\neq\pm\sqrt{4}$

$\therefore D=(-1,0)\cup(1,\infty)-\{\sqrt{4}\}$

$D=(-1,0)\cup(1,2)\cup(2,\infty)$.

61. f(x + y) = f(x) + f(y). Let $f(\alpha)=m\alpha$

f(1) = 7; $\therefore$ m = 7, f(x) = 7x

$\sum_{r=1}^{n} f(r)=7\sum_{1}^{n} r=\frac{7n(n+1)}{2}$

62. $y=x+\frac{1}{x}$ or $\frac{dy}{dx}=1-\frac{1}{x^2}$

For max. or min., $1-\frac{1}{x^2}=0\Rightarrow x=\pm 1$

$\frac{d^2y}{dx^2}=\frac{2}{x^3}\Rightarrow\left(\frac{d^2y}{dx^2}\right)_{x=2}=2(+\text{ve minima})$

Therefore x = 1

63. Let β be the inclination of the plane to the horizontal and u be the velocity of projection of the projectile

$R_1=\frac{u^2}{g(1+\sin\beta)}$ and $R_2=\frac{u^2}{g(1-\sin\beta)}$

$\frac{1}{R_1}+\frac{1}{R_2}=\frac{2g}{u^2}$ or $\frac{1}{R_1}+\frac{1}{R_2}=\frac{2}{R}\left[\because R=\frac{u^2}{g}\right]$

Therefore, R_1, R, R_2 are in H.P.

64. $\Sigma x = 170,\ \Sigma x^2 = 2830$ increase in $\Sigma x = 10$, then

$\Sigma x' = 170 + 10 = 180$

Increase in $\Sigma x^2 = 900 - 400 = 500$ then

$\Sigma x'^2 = 2830 + 500 = 3330$

Variance = $\frac{1}{n}\Sigma x'^2 - \left(\frac{1}{n}\Sigma x'\right)^2$

$= \frac{1}{15}\times 3330 - \left(\frac{1}{15}\times 180\right)^2 = 222 - 144 = 78.$

65. As for given question two cases are possible.

(i) Selecting 4 out of first five question and 6 out of remaining 8 question = ${}^5C_4 \times {}^8C_6$ = 140 choices.

(ii) Selecting 5 out of first five question and 5 out of remaining 8 questions = ${}^5C_5 \times {}^8C_5$ = 56 choices.

Therefore, total number of choices = 140 + 56 = 196.

66. $A^2 = \begin{bmatrix} \alpha & \beta \\ \beta & \alpha \end{bmatrix} = \begin{bmatrix} a & b \\ b & a \end{bmatrix}\begin{bmatrix} a & b \\ b & a \end{bmatrix}$

$\alpha = a^2 + b^2;\ \beta = 2ab$

67. No.of ways in which 6mm can be arranged at a round table = (6 - 1)!

Now women can be arranged in 6! ways.

Total number of ways = $6! \times 5!$

68. No option satisfied wrong.

A = (7, -4, 7), B = (1, -6, 10), C = (-1, -3, 4) and D = (5, -1, 5)

$AB = \sqrt{(7-1)^2 + (-4+6)^2 + (7-10)^2} = \sqrt{36+4+9} = 7$

Similarly BC = 7, CD = $\sqrt{41}$, DA = $\sqrt{17}$

69. $(\vec{u} + \vec{v} - \vec{w}).(\vec{u}\times\vec{v} - \vec{u}\times\vec{w} - \vec{v}\times\vec{v} + \vec{v}\times\vec{w})$

$(\vec{u} + \vec{v} - \vec{w}).(\vec{u}\times\vec{v} - \vec{u}\times\vec{w} + \vec{v}\times\vec{w}) = \frac{\vec{u}.(\vec{u}\times\vec{v})}{0}$

$-\frac{\vec{u}.(\vec{u}\times\vec{w})}{0} + \vec{u}.(\vec{v}\times\vec{w}) + \frac{\vec{v}.(\vec{u}\times\vec{v})}{0} - \vec{v}.(\vec{u}\times\vec{w})$

$+\frac{\vec{v}.(\vec{v}\times\vec{w})}{0} - \vec{w}.(\vec{u}\times\vec{v}) + \frac{\vec{w}.(\vec{u}\times\vec{w})}{0} - \frac{\vec{w}.(\vec{u}\times\vec{w})}{0} = \vec{u}.(\vec{v}\times\vec{w}) - \vec{v}.(\vec{u}\times\vec{w}) - \vec{w}.(\vec{u}\times\vec{v})$

$= [\vec{u}\vec{v}\vec{w}] + [\vec{v}\vec{w}\vec{u}] - [\vec{w}\vec{u}\vec{v}] = \vec{u}.(\vec{v}\times\vec{w})$

70. $\sin^{-1} x = 2\sin^{-1} a$

$-\frac{\pi}{2} \le \sin^{-1} x \le \frac{\pi}{2};\ \therefore -\frac{\pi}{2} \le 2\sin^{-1} a \le \frac{\pi}{2}$ $\quad -\frac{\pi}{4} \le \sin^{-1} a \le \frac{\pi}{4}$ or $\frac{-1}{\sqrt{2}} \le a \le \frac{1}{\sqrt{2}}$

$\therefore |a| \le \frac{1}{\sqrt{2}}$ (As $\frac{1}{\sqrt{2}} > \frac{1}{2}$). Out of given four option no one is absolutely correct but (c) could be taken into consideration. $\rightarrow |a| \le \frac{1}{\sqrt{2}}$ is correct, if $a < \frac{1}{\sqrt{2}}$ is taken as correct then it domain satisfy for $a = \frac{1}{\sqrt{3}}$ but equation is satisfied. $\frac{1}{\sqrt{2}} > \frac{1}{\sqrt{3}} > \frac{1}{2}$

71. Eq. of planes be $\frac{x}{a}+\frac{y}{b}+\frac{z}{c}=1$ & $\frac{x}{a_1}+\frac{y}{b_1}+\frac{z}{c_1}=1$ ($\perp$r distance on plane from origin is same.)

$$\left|\frac{-1}{\sqrt{\frac{1}{a^2}+\frac{1}{b^2}+\frac{1}{c^2}}}\right|=\left|\frac{-1}{\sqrt{\frac{1}{a_1^2}+\frac{1}{b_1^2}+\frac{1}{c_1^2}}}\right|$$

$$\therefore \Sigma\frac{1}{a^2}-\Sigma\frac{1}{a_1^2}=0$$

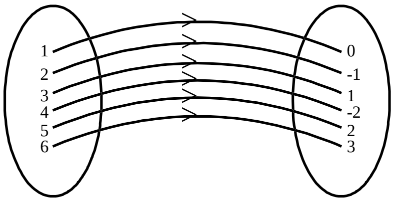
72. $\left(\frac{1+i}{1-i}\right)^x=1 \Rightarrow \left[\frac{(1+i)}{1-i^2}\right]^x=1$

$$\left(\frac{1+i^2+2i}{1+1}\right)^x=1 \Rightarrow (i)^x=1; \ \therefore x=4n; \ n\in 1^+$$

73. $f: N \to 1$

f(1) = 0, f(2) = -1, f(3) = -1, f(4) = -2,

f(5) = 2, and f(6) = -3 so on.



In this type of function every element of set A has unique image in set B and there is no element left in set B. Hence f is one-one and onto function.

74. $f(x) = ax^2 + bx + c$

$f(1) = f(-1) \Rightarrow a + b + c = a - b + c$ or $b = 0$

$\therefore$ $f(x) = ax^2 + c$ or $f'(x) = 2ax$

Now $f'(a)$; $f'(b)$; and $f'(c)$ are 2a(a); 2a(b); 2a(c). If a, b, c are in A.P. then $f'(a)$; $f'(b)$ and $f'(c)$ are also in A.P.

75. $\frac{1}{1.2}-\frac{1}{2.3}+\frac{1}{3.4}$ .... ∞

Let $T_n=\frac{1}{n(n+1)}=\left(\frac{1}{n}-\frac{1}{n+1}\right)$

$S = T_1 - T_2 + T_3 - T_4 + T_5$ .... ∞

$$=\left(\frac{1}{1}-\frac{1}{2}\right)-\left(\frac{1}{2}-\frac{1}{3}\right)+\left(\frac{1}{3}-\frac{1}{4}\right)-\left(\frac{1}{4}-\frac{1}{5}\right)..........$$

$$=1-2\left[\frac{1}{2}-\frac{1}{3}+\frac{1}{4}-\frac{1}{5}...........\infty\right]$$

$$=1-2[-\log(1+1)+1]=2\log 2-1=\log\left(\frac{4}{e}\right)$$

AIEEE 2003 KEY

Physics And Chemistry		37.	C	75.	D	113.	A	***Mathematics***		38.	D
		38.	D	76.	B	114.	d	1.	A	39.	A
1.	B	39.	B	77.	C	115.	B	2.	A	40.	B
2.	A	40.	D	78.	B	116.	D	3.	A	41.	D
3.	B	41.	C	79.	B	117.	A	4.	B	42.	C
4.	A	42.	A	80.	D	118.	B	5.	B	43.	C
5.	D	43.	D	81.	C	119.	B	6.	D	44.	A
6.	A	44.	A	82.	C	120.	B	7.	D	45.	A
7.	A	45.	B	83.	B	121.	C	8.	C	46.	NONE
8.	D	46.	A	84.	D	122.	A	9.	C	47.	B
9.	C	47.	A	85.	D	123.	B	10.	D	48.	C
10.	B	48.	B	86.	C	124.	A	11.	D	49.	D
11.	A	49.	B	87.	A	125.	A	12.	A	50.	D
12.	C	50.	C	88.	D	126.	A	13.	A	51.	D
13.	C	51.	C	89.	A	127.	D	14.	C	52.	B
14.	D	52.	D	90.	A	128.	A	15.	D	53.	B
15.	C	53.	B	91.	B	129.	B	16.	B	54.	D
16.	A	54.	D	92.	C	130.	C	17.	C	55.	B
17.	A	55.	C	93.	C	131.	D	18.	B	56.	C
18.	A	56.	B	94.	B	132.	A	19.	D	57.	C
19.	C	57.	B	95.	C	133.	D	20.	A	58.	D
20.	B	58.	A	96.	A	134.	B	21.	D	59.	C
21.	C	59.	A	97.	C	135.	D	22.	D	60.	A
22.	D	60.	A	98.	C	136.	C	23.	D	61.	A
23.	D	61.	D	99.	B	137.	C	24.	D	62.	C
24.	B	62.	B	100.	A	138.	D	25.	A	63.	A
25.	D	63.	C	101.	A	139.	B	26.	D	64.	B
26.	A	64.	C	102.	A	140.	A	27.	A	65.	C
27.	D	65.	B	103.	D	141.	B	28.	A	66.	C
28.	NONE	66.	B	104.	B	142.	B	29.	A	67.	A
29.	C	67.	A	105.	A	143.	B	30.	B	68.	C
30.	B	68.	D	106.	D	144.	A	31.	D	69.	C
31.	D	69.	B	107.	A	145.	B	32.	B	70.	C
32.	A	70.	C	108.	D	146.	B	33.	A	71.	A
33.	D	71.	C	109.	A	147.	A	34.	D	72.	B
34.	C	72.	D	110.	B	148.	A	35.	C	73.	D
35.	C	73.	B	111.	B	149.	A	36..	D	74.	B
36.	A	74.	D	112.	D	150.	D	37.	D	75.	A